which are close to the boundary of the disk. The relative dimensions of the disks were $r / R = 0.23$, $H / R = 0.4$ and $r / R = 0.3$, $H / R = 0.6$, respectively. The order of the normal system was chosen to be 22, the duration of the computations was about 30 minutes. The closeness of the obtained solution to the exact one was estimated from the relative error in the realization of the boundary conditions, being 4% in the first case and 2.5% in the second case. For $N = 6$, for the same disposition of the holes and the same accuracy, the duration of the computation increased to 42 minutes.

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Translated by E. D.

UDC 539.3

ON A PROBLEM OF NONLINEAR DEFORMATION OF A CYLINDRICAL SHELL

PMM Vol. 38, № 5, 1974, pp. 942-944
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(Received May 29, 1973)

Problems of the axisymmetric deformation of elastic thin-walled shells of revolution, taking into account the finiteness of the displacements, have been examined sufficiently completely up to now for spherical type shells. Thus, numerical methods have been developed in [1-4] and solutions have been obtained for domes of diverse geometry under various external effects. It is shown below in the example of a long cylindrical shell that equilibrium modes of the rubber type of a flexible rod appear for shells of revolution whose Gaussian curvature is almost zero, under definite effects.

Let a cylindrical shell of thickness h and radius R (Fig. 1) be compressed uniformly by longitudinal stress resultants N and heated to the temperature $t(x) = {}^1/_2\ T \operatorname{sign}(x)$,

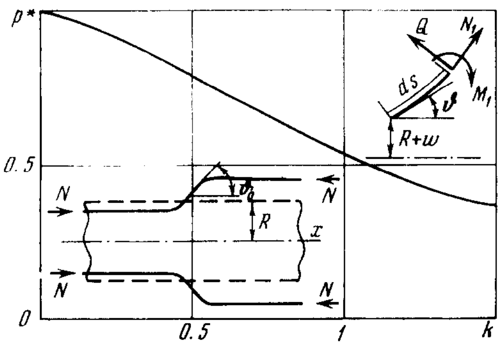

Fig. 1

which is constant along the thickness. We seek symmetric equilibrium modes relative to the shell axis. We use a linear dependence between the stresses and strains by assuming the strains small compared to unity [5]. Under these assumptions, the strained middle surface is skew-symmetric relative to the section $x = 0$ to the accuracy of small quantities on the order of $\sqrt{h/R}$. The strain potential energy is determined by the bending moment $M_1 = -D d\vartheta / ds$ and the circumferential stress resultant $N_2 = Ehw / R$. Here s is the arclength of strain middle surface meridian, ϑ, w are the angular displacement and deflection measured along the normal to the initial surface, E, ν are the elastic modulus and Poisson's ratio, respectively, and $D = Eh^3 / [12(1 - \nu^2)]$.

Let us introduce the dimensionless quantities

$$f = \beta(w - w_0), \qquad \xi = \beta s, \qquad p = N / N^* = {}^1/_2\, NR / \sqrt{DEh}$$

where

$$\beta = [12(1 - \nu^2) / (hR)^2]^{1/4}, \qquad w_0 = {}^1/_2\, \alpha RT + \nu NR / Eh$$

(α is the coefficient of temperature expansion, N^* is the Euler critical force).

We consider the problem of the shell equilibrium modes as a problem of the extremum of the functional

$$I = \int_0^\infty [\vartheta'^2 + f^2 - 4p(1 - \cos\vartheta)]\, d\xi$$

under the condition $f' = \sin\vartheta$ (the prime denotes differentiation with respect to ξ). By using Lagrange multipliers, we obtain the equation of the problem and the natural boundary conditions. By appending the kinematic conditions (taking symmetry into account), we formulate the following boundary value problem:

$$(2p \operatorname{tg}\vartheta + \vartheta'' / \cos\vartheta)' + f = 0, \qquad f' - \sin\vartheta = 0 \qquad (0 \leqslant \xi < \infty) \tag{1}$$
$$\vartheta'(0) = 0, \qquad f(0) = -k \; (k = {}^1/_2\, \alpha\beta RT), \qquad f(\infty) = \vartheta(\infty) = 0$$

Let us note that the system (1) can be represented in the form obtained in [6] by introducing the function $\psi = 2(N_1 \sin\vartheta + Q\cos\vartheta) / N^*$, or taking account of the appropriate physical and geometric relationships (under the assumptions made earlier). This system can be reduced to integrable form by simple manipulations. Taking account of the condition at infinity, the first integrals of the system are

$$2\vartheta'' \sin\vartheta - (\vartheta'^2 - f^2)\cos\vartheta + 4p(1 - \cos\vartheta) = 0 \tag{2}$$
$$\vartheta'^2 - f^2 - 2\sin\vartheta \int_\xi^\infty f\, d\xi + 4p(1 - \cos\vartheta) = 0$$

Let ϑ_0 denote the angular displacement ϑ at $\xi = 0$. We note that the solution corresponding to the value $\vartheta_0 = \pi$ can hold for $k^2 = 8p$, which permits the assumption of

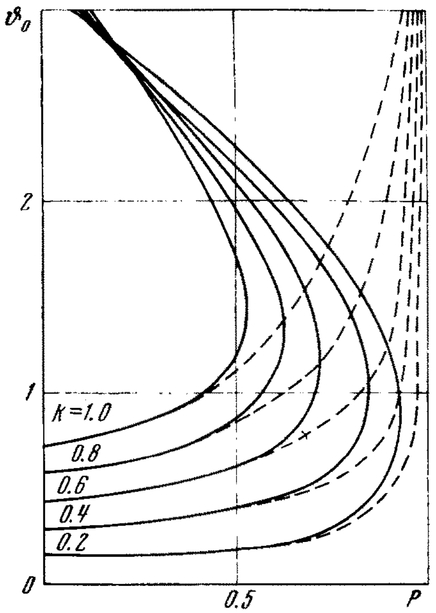

Fig. 2

the existence of equilibrium modes characterizing sufficiently large angular displacements. For $\xi = 0$ the first of the relationships (2) permits efficient use of the method of reduction to a Cauchy problem for the numerical solution because the initial conditions for given values of the parameters k and p can be expressed explicitly in terms of ϑ_0. The problem is reduced to seeking that value of ϑ_0 for which the solution of the Cauchy problem is damped out for large values of ξ. Taking the damping into account, the linearized equations

$$F_1 = f'' + \lambda f' + f, \qquad \lambda = \sqrt{2(1-p)}$$
$$F_2 = \vartheta'' + \lambda\vartheta' + \vartheta, \qquad \xi \geqslant \xi^*$$

can be written instead of (1) starting with some $\xi = \xi^*$. Hence, the requirement for damping of the solution can be formulated as a conjugate condition for the solution with damping

$$F_i(\xi^*) = 0, \qquad i = 1,2 \tag{3}$$

Such values of ϑ_0 were selected during the computation that the solutions obtained for the Cauchy problem would satisfy conditions (3) for different values of the parameters k and p, and in this sense would be solutions of (1). The value of ξ^* was determined during the computation from the conditions of complying with (3) with given accuracy. The dependences $\vartheta_0 = \vartheta_0(p)$ obtained for fixed k are presented in Fig. 2 (solid lines). The essential difference from the dependence $\vartheta_0 = k / \lambda$ (dashed lines) which holds in the linear formulation is that for the same value of p the existence of both a stable equilibrium mode similar to that described by linear theory, and an "inverted" (unstable) equilibrium mode is possible. For definite values of the parameters $k = k^*$, $p = p^*$, the existence of adjacent axisymmetric equilibrium modes is possible, i.e. buckling "in the large" occurs. The curve of the dependence of the critical value of the axial load parameter p^* on the value of the heating parameter k is presented in Fig. 1. For $p < 1/3$ the mentioned phenomenon does not hold for any values of the heating parameter.

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Translated by M. D. F.

UDC 539.3

ON THE SOLVABILITY OF NONLINEAR EQUATIONS FOR A SYMMETRICALLY LOADED NONSHALLOW SPHERICAL DOME

PMM Vol. 38, № 5, 1974, pp. 944-946
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(Received September 3, 1973)

Existence of the generalized solution in the problem of equilibrium of the isotropic elastic nonshallow spherical dome with rigidly held edge and subjected to axisymmetric deformation is proved by the method presented in [1]. The topological characteristic of the problem, i.e. the vector field rotation is computed. The solvability of nonlinear equations for nonshallow shells of revolution subjected to symmetric load was investigated in [2, 3]. However dome-shaped shells were not considered there.

1. Fundamental relationships. We consider the following version of relationships of the nonlinear theory of nonshallow symmetrically loaded shells of revolution:

$$\begin{aligned}
&T_1(\varepsilon_j) = K(\varepsilon_1 + \nu\varepsilon_2), \quad M_1 = D(\varkappa_1 + \nu\varkappa_2) \quad 1 \rightleftarrows 2 \qquad (1.1)\\
&\varepsilon_1 = vA'(AB)^{-1} + wR_1^{-1}, \quad \varepsilon_2 = v'B^{-1} + wR_2^{-1} + \psi^2 2^{-1}\\
&\varkappa_1 = -\psi A'(AB)^{-1}, \quad \varkappa_2 = -\psi' B^{-1}, \quad \psi = w'B^{-1} - vR_2^{-1}\\
&T_{12} = M_{12} = \varepsilon_{12} = \varkappa_{12} = 0\\
&K = 2hE(1-\nu^2)^{-1}, \qquad D = 2h^3[3(1-\nu^2)]^{-1}\llcorner E
\end{aligned}$$

where T_i and T_{12} are tangential stresses; ε_i and ε_{12} are the tensile and shear strains, respectively; M_i and M_{12} are, respectively, the bending moment and the torque; $\varkappa_i$ and $\varkappa_{12}$ are changes of curvature R_i^{-1} of the shell middle plane s^*; v and w are, respectively, the tangential and normal displacement of the shell middle plane s^*; A^2, B^2, $2C = 0$ are coefficients of the first quadratic form of surface s^*; $E > 0$ is the Young modulus; $0 < \nu < 1/2$ is the Poisson ratio, and $2h$ is the thickness of the shell. A prime superscript denotes differentiation with respect to parameter β.

The analysis of a spherical dome is conveniently carried out in spherical coordinates un which $A = \rho \sin\beta$, $\beta \in [0, b]$, $B = R_i = \rho$, where ρ is the radius of the shell middle plane s^*. For convenience we set $\rho \equiv 1$. The substitution $v = w' - \psi$ eliminates v from all formulas. We introduce the notation

$$e_1(w) = w' \operatorname{ctg}\beta + w, \qquad e_2(w) = w'' + w \qquad (1.2)$$

The equation of the shell equilibrium is determined by the Lagrange principle which